

Logarithmic Functions: Problems and Solutions

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Instructions

Solve the following problems using the properties of logarithmic functions. The solutions are provided below each problem.

Problems and Solutions

Problem 1

Simplify the expression using logarithmic properties:

$$\log_5(25) + \log_5(5)$$

Solution:

$$\log_5(25) + \log_5(5) = \log_5(25 \times 5) = \log_5(125) = \log_5(5^3) = 3$$

Problem 2

Solve for x using the properties of logarithms:

$$\log_2(x) + \log_2(8) = 4$$

Solution:

$$\log_2(x \times 8) = 4 \Rightarrow \log_2(8x) = 4 \Rightarrow 8x = 2^4 = 16 \Rightarrow x = 2$$

Problem 3

Express as a single logarithm:

$$\log_3(81) - \log_3(9)$$

Solution:

$$\log_3\left(\frac{81}{9}\right) = \log_3(9) = \log_3(3^2) = 2$$

Problem 4

Use the change of base formula to evaluate the logarithm:

$$\log_4(16)$$

Solution:

$$\log_4(16) = \frac{\log_{10}(16)}{\log_{10}(4)} = \frac{\log_{10}(2^4)}{\log_{10}(2^2)} = \frac{4 \log_{10}(2)}{2 \log_{10}(2)} = 2$$

Problem 5

Simplify the expression:

$$\frac{1}{3} \log_2(8) + \frac{1}{2} \log_2(4)$$

Solution:

$$\frac{1}{3} \log_2(8) + \frac{1}{2} \log_2(4) = \frac{1}{3} \times 3 + \frac{1}{2} \times 2 = 1 + 1 = 2$$

Problem 6

Solve for y in the equation:

$$\log_{10}(y) = \log_{10}(5) + \log_{10}(2)$$

Solution:

$$\log_{10}(y) = \log_{10}(5 \times 2) = \log_{10}(10) = 1 \Rightarrow y = 10$$

Problem 7

Express the logarithmic expression as a sum or difference of logarithms:

$$\log_b \left(\frac{x^2 \cdot y^3}{z} \right)$$

Solution:

$$\log_b \left(\frac{x^2 \cdot y^3}{z} \right) = \log_b(x^2) + \log_b(y^3) - \log_b(z) = 2 \log_b(x) + 3 \log_b(y) - \log_b(z)$$

Problem 8

Use the properties of logarithms to simplify:

$$\log_7(7^4) - 2 \log_7(7)$$

Solution:

$$\log_7(7^4) - 2 \log_7(7) = 4 \log_7(7) - 2 \log_7(7) = 4 \times 1 - 2 \times 1 = 4 - 2 = 2$$

Problem 9

Solve the logarithmic equation for x :

$$2 \log_3(x) = \log_3(9)$$

Solution:

$$2 \log_3(x) = \log_3(9) \Rightarrow \log_3(x^2) = \log_3(9) \Rightarrow x^2 = 9 \Rightarrow x = 3 \text{ or } x = -3$$

Problem 10

Express the following logarithmic expression using the change of base formula:

$$\log_5(50)$$

Solution:

$$\log_5(50) = \frac{\log_{10}(50)}{\log_{10}(5)}$$