Exponential Functions: Problems and Solutions

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Instructions

Solve the following problems related to exponential functions. Solutions are provided after each problem.

Problems and Solutions

Problem 1

Solve the exponential equation for x:

Solution:

 $2^x = 16 = 2^4 \quad \Rightarrow \quad x = 4$

 $2^x = 16$

Problem 2

Simplify the following exponential expression:

 $3^x \cdot 3^{2x}$

 $3^x \cdot 3^{2x} = 3^{x+2x} = 3^{3x}$

Solution:

Problem 3

Solve the exponential equation for x:

$$5^{2x} = 125$$

Solution:

$$5^{2x} = 125 = 5^3 \quad \Rightarrow \quad 2x = 3 \quad \Rightarrow \quad x = \frac{3}{2}$$

Problem 4

Express the following exponential expression as a single exponent:

 $2^{x+1} \cdot 2^{2x}$

Solution:

$$2^{x+1} \cdot 2^{2x} = 2^{(x+1)+2x} = 2^{3x+1}$$

Problem 5

The population of a certain species of bacteria doubles every 3 hours. If the initial population is 500, find the population after 9 hours.

Solution: Let the population be $P(t) = P_0 \cdot 2^{t/3}$, where t is time in hours and $P_0 = 500$.

$$P(9) = 500 \cdot 2^{9/3} = 500 \cdot 2^3 = 500 \cdot 8 = 4000$$

The population after 9 hours is 4000.

Problem 6

Solve the exponential equation for x:

$$e^{2x} = e^5$$

Solution:

$$e^{2x} = e^5 \quad \Rightarrow \quad 2x = 5 \quad \Rightarrow \quad x = \frac{5}{2} = 2.5$$

Problem 7

Simplify the following expression using the properties of exponents:

$$\left(3^{x+1}\right)^2$$

Solution:

$$\left(3^{x+1}\right)^2 = 3^{2(x+1)} = 3^{2x+2}$$

Problem 8

The half-life of a radioactive substance is 6 years. If the initial amount is 100 grams, how much remains after 18 years?

Solution: Let the remaining amount be $A(t) = A_0 \cdot \left(\frac{1}{2}\right)^{t/6}$, where t is time in years and $A_0 = 100$ grams.

$$A(18) = 100 \cdot \left(\frac{1}{2}\right)^{18/6} = 100 \cdot \left(\frac{1}{2}\right)^3 = 100 \cdot \frac{1}{8} = 12.5 \text{ grams}$$

The remaining amount after 18 years is 12.5 grams.

Problem 9

Express the following exponential expression in logarithmic form:

$$7^x = 49$$

Solution:

$$7^x = 49 = 7^2 \implies x = 2 \text{ so, } \log_7(49) = 2$$

Problem 10

Solve the exponential equation for x:

$$10^{2x+1} = 1000$$

Solution:

$$10^{2x+1} = 1000 = 10^3 \quad \Rightarrow \quad 2x+1 = 3 \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1$$